Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

J80-~~[illegible]~~ 239

High Mach Number Dynamic Stability of Pointed Cones at Small Angles of Attack

00001
20009
20017

M. Khalid* and R. A. East†
University of Southampton, Hants, England

Nomenclature

c = pointed cone length
C_m = pitching moment coefficient
C_{m_α} = static stability derivative,

$$\left(\frac{\partial C_m}{\partial \alpha}\right)_{\substack{q=0,\ \ddot{z}=0 \\ \dot{\alpha}=0,\ \alpha_e\to\alpha}}$$

$C_{m_{\dot\theta}}$ = dynamic stability derivatives,

$$\left(\frac{\partial C_m}{\partial qc/2v_\infty}\right)_{\substack{z=0,\ \dot{\alpha}=0 \\ \alpha_e\to\alpha,\ q\to 0}} = (C_{m_q}+C_{m_{\dot\alpha}})$$

C_p = pressure coefficient, $2/\gamma M_\infty^2\,(p/p_\infty - 1)$
$K\lambda_1$ = $\gamma M_\infty^2 \tan^2\theta_0$
l' = moment arm
M = Mach number
p = local flow pressure
q = angular velocity of body about pitch axis, $x = x_0$ (positive in nose up direction)
R_e = Reynolds number based on the cone length
$R(x)$ = local cross-sectional radius of the body
S = base surface area
V = velocity
x = local axial position measured from the apex of the pointed cone
x_0 = coordinate of pitch axis of the body
$\ddot{z}$ = vertical acceleration of body pitch axis
α = angle of incidence of the body in general motion
η, η', η'' = Stone-Kopal pressure perturbation coefficients
$\Delta\epsilon_p$ = change in angle of inclination of tangent to (projected) streamline (pathline) of particular fluid element between $(x, R(x), \phi)$ and the reference point on body surface, measured in osculating plane of streamline
θ_0 = cone semi-angle
ϕ = (angular) cylindrical polar coordinate, fixed relative to moving body
γ = ratio of principle specific heats of gas, C_p/C_v

Subscripts

α = angle of attack condition
e = effective
x = local condition
(x,t) = local conditions with respect to time
1 = conditions at the pointed cone surface
∞ = freestream conditions
ref = conditions at the reference point

Received Feb. 11, 1980.
Index categories: Aerodynamics; Nonsteady Aerodynamics; Supersonic and Hypersonic Flow.
*Research Student; presently Research Engineer, Canadair Ltd., Montreal, Canada.
†Senior Lecturer, Dept. of Aeronautics and Astronautics.

An analytical method has been presented previously[1,2] for calculating the pitching stability derivatives $-C_{m_\alpha}$ and $-C_{m_{\dot\theta}}$ $[= -(C_{m_q}+C_{m_{\dot\alpha}})]$ of pointed and blunt slender cones performing small amplitude oscillations about a zero mean angle of attack in hypersonic flow. The essence of this method is to perturb a previously known steady flow solution by employing the modified shock-expansion theory due to Eggers and Savin.[3] The inviscid pressure distribution $p(x,t)$ on the oscillating body may be written as the sum of the steady pressure distribution $p(x)$ and a nonsteady perturbation $p'(x,t)$. Thus,

$$p(x,t) = p(x) + p'(x,t) \tag{1}$$

where $p'(x,t)$ is determined from the Prandtl-Meyer flow relationship along a streamline as

$$p'(x,t) = \frac{\rho(x)\,V(x)^2}{\sqrt{M(x)^2-1}}\Delta\epsilon_p(x) \tag{2}$$

where $\Delta\epsilon_p(x)$, the change in the inclination of a streamline due to the pitching motion involving $\ddot{z}$ and q only, is given by

$$\Delta\epsilon_p(x)_{\ddot{z},q} = \frac{\sin\phi}{V_\infty}\left(\frac{\ddot{z}}{V_\infty} - 2xq\cos\theta_0\right) \tag{3}$$

The pressure coefficient on the oscillating body is

$$C_p(x,t) = \frac{2}{\gamma M_\infty^2}\left\{\frac{p(x,t)}{p_\infty} - 1\right\} \tag{4}$$

where, generally, for a body oscillating about a mean angle of attack α_e,

$$\frac{p(x,t,\alpha_e)}{p_\infty} = \frac{p(x,t,\alpha_e)}{p(x,\alpha_e)}\,\frac{p(x,\alpha_e)}{p(0,\alpha_e)}\,\frac{p(0,\alpha_e)}{p_1}\,\frac{p_1}{p_\infty} \tag{5}$$

In Eq. (5), $p(x,\alpha_e)$ is the pressure at a general point x on the body at angle of attack α_e in steady flow, $p(0,\alpha_e)$ is the corresponding surface pressure at a suitably defined reference point which for a pointed cone is immediately downstream of the bow shock $(x=0)$, and p_1 is the pressure on the unyawed pointed cone at the reference point.

For pointed cones at small angle of attack α_e, Stone,[4] Kopal,[5] and Taylor and Maccoll[6] have shown that

$$p(0,\alpha_e)/p_1 = 1 - \eta\alpha_e\sin\phi + \alpha_e^2(\eta' - \eta''\cos 2\phi) \tag{6}$$

where η, η', and η'' are numerical pressure perturbation coefficients which may be obtained from Refs. 4 and 5.

Using Eqs. (2) and (5), the pressure on the oscillating pointed cone at a mean angle of attack α_e may be expressed as

$$\frac{p(x,t,\alpha_e)}{p_\infty} = \frac{p_1}{p_\infty}\left(1 + \frac{\gamma M_1^2\Delta\epsilon_p}{\sqrt{M_1^2-1}}\right)\{1 - \eta\alpha_e\sin\phi + \alpha_e^2(\eta' - \eta''\cos 2\phi)\} \tag{7}$$

where

$$\alpha_e = \theta - q(x\sec\theta_0 + x_0)/V_\infty \tag{8}$$

The unsteady pressure coefficient is written as

$$C_p(x,t,\alpha_e) = \frac{2}{\gamma M_\infty^2}\left\{\frac{p(x,t,\alpha_e)}{p_\infty} - 1\right\} = \frac{2}{\gamma M_\infty^2}\Bigg[\left(\frac{p_1}{p_\infty} - 1\right) + \frac{p_1}{p_\infty}\Bigg\{\frac{\gamma M_1^2}{\sqrt{M_1^2-1}}\Delta\epsilon_p - \eta\alpha_e\sin\phi - \frac{\gamma M_1^2}{\sqrt{M_1^2-1}}\Delta\epsilon_p\eta\alpha_e\sin\phi + \alpha_e^2(\eta' - \eta''\cos2\phi) + \frac{\gamma M_1^2}{\sqrt{M_1^2-1}}\Delta\epsilon_p\alpha_e^2(\eta' - \eta''\cos2\phi)\Bigg\}\Bigg] \tag{9}$$

The pitching moment coefficient is

$$C_m = \frac{1}{Sc}\int_0^{2\pi}\int_0^c C_p(x,t,\alpha_e)\,l'\sin\phi\,d\phi\,dx$$

where the moment arm $l' = x\sec^2\theta_0 - x_0$,

$$C_{m_\alpha} = \left(\frac{\partial C_m}{\partial \alpha}\right)_{\substack{q=0,\ \ddot{z}=0,\\ \dot{a}=0,\ \alpha_e\to\alpha}} \qquad C_{m_{\dot\theta}} = \frac{2V_\infty}{c}\left(\frac{\partial C_m}{\partial qc/2V_\infty}\right)_{\substack{\ddot{z}=0,\ \alpha=0\\ \alpha_e\to\alpha,\ q\to 0}} \tag{10}$$

$$C_{m_\alpha} = \frac{1}{Sc}\int_0^c\int_0^{2\pi}\frac{\partial C_p}{\partial\alpha}(x\sec^2\theta_0 - x_0)x\tan\theta_0\sin\phi\,d\phi\,dx \tag{11}$$

$$C_{m_{\dot\theta}} = \frac{2V_\infty}{Sc^2}\int_0^c\int_0^{2\pi}\frac{\partial C_p}{\partial q}(x\sec^2\theta_0 - x_0)x\tan\theta_0\sin\phi\,d\phi\,dx \tag{12}$$

Using the expression for $C_p(x,t,\alpha_e)$ in Eq. (9)

$$\frac{\partial C_p}{\partial\alpha} = -\frac{2}{\gamma M_\infty^2}\frac{p_1}{p_\infty}\eta\sin\phi \tag{13}$$

$$\frac{\partial C_p}{\partial q} = \frac{2}{\gamma M_\infty^2}\frac{p_1}{p_\infty}\Bigg\{\frac{\eta}{V_\infty}(x\sec\theta_0 + x_0)\sin\phi - \frac{2\gamma M_1^2}{\sqrt{M_1^2-1}}x\frac{\cos\theta_0}{V_\infty}\sin\phi - \frac{2\alpha^2 M_1^2}{V_\infty\sqrt{M_1^2-1}}(\eta' - \eta''\cos2\phi)x\cos\theta_0\sin\phi\Bigg\} \tag{14}$$

Terms in Eqs. (13) and (14), which will constitute zero contribution when substituted back in Eqs. (11) and (12), respectively, and integrated, have been omitted.

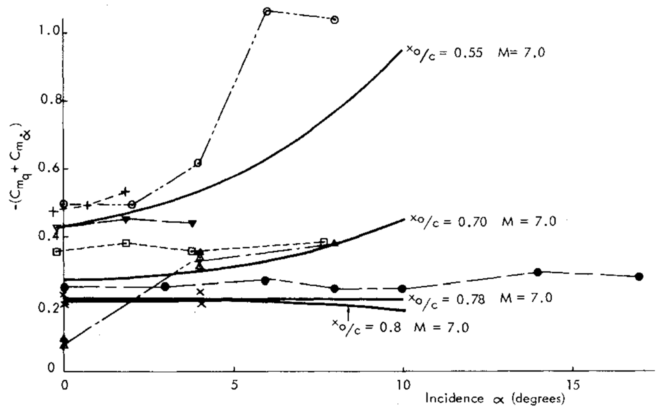
The resulting derivatives are

$$C_{m_\alpha} = \frac{2}{\gamma M_\infty^2}\frac{p_1}{p_\infty}\cot\theta_0\eta\left(\frac{\sec^2\theta_0}{3} - \frac{1}{2}\frac{x_0}{c}\right) \tag{15}$$

$$C_{m_{\dot\theta}} = \frac{4}{\gamma M_\infty^2}\frac{p_1}{p_\infty}\cos\theta_0\Bigg\{\eta\left(\frac{\sec^3\theta_0}{4} + \frac{1}{3}\frac{x_0}{c}\sec^2\theta_0 - \frac{1}{3}\frac{x_0}{c}\times\sec\theta_0 - \frac{1}{2}\left(\frac{x_0}{c}\right)^2\right) - 2\cos\theta_0\frac{\gamma M_1^2}{\sqrt{M_1^2-1}}\times\left(1 + \alpha^2\eta' + \frac{\alpha^2}{2}\eta''\right)\left(\frac{\sec^2\theta_0}{4} - \frac{x_0}{3c}\right)\Bigg\} \tag{16}$$

Results and Discussions

A comparison of the present stability derivatives with the corresponding expressions obtained in Refs. 2 and 7 shows that the static stability remains unchanged after the introduction of the second-order pressure perturbation coefficients η' and η''.



	x_0/c	R_{e_c}	M	$\bar{\chi} = (M^3/\sqrt{R_{e_c}})$	
⊙	0.55	9.6×10^6	8.0	1.65×10^{-1}	SCHUELER (8(i))
×	0.55	0.5×10^6	14	3.88	" " (8(ii))
▵	0.55	0.17×10^6	20.2	20.0	" " (8(iii))
⊡	0.60	7.32×10^5	10.18	1.23	MARSHALL (9(i))
+	0.60	5.17×10^6	10.18	4.64×10^{-1}	" " (9(ii))
▿	0.60	3×10^6	10.18	6.09×10^{-1}	" " (9(iii))
●	0.70	2.17×10^6	7.0	2.32×10^{-1}	QASRAWI (10)

—— THEORY PRESENT WORK (M = 7) ~ EQUATION 16

Fig. 1 Dynamic stability derivatives of 10-deg semi-angle cones vs angle of attack.

The extent to which the dynamic stability derivative is affected by the angle of attack is dependent on the axis of oscillation position. Theoretical curves plotted in Fig. 1 for a 10 deg semi-angle pointed cone show that a steady increase in dynamic stability is predicted with increasing forward movement of the axis of oscillation position. Theory shows that at an axis of oscillation position of $x_0/c = 0.78$, the dynamic stability is virtually unaffected with the angle of attack change. A cone oscillating at a station aft of this position is dynamic destabilized by increasing angle of attack.

Available experimental data for the variation of the pitch damping derivative $-(C_{m_q} + C_{m_{\dot\alpha}})$ with mean angle of attack for 10 deg semi-angle pointed cones are compared with the predictions of the theory for the range of axis of oscillation given by $0.55 \leq x_0/c \leq 0.8$ in Fig. 1. It is noted that the results of the present theory are for inviscid flow, whereas many of the experimental results are affected by viscous effects. It is well established that nonsteady transition point movement can lead to an increase in damping and a decrease in stiffness for pointed slender cones at zero angle of attack at hypersonic speeds. A measure of the likely influence of viscous effects is provided by the viscous interaction parameter $\bar{\chi} = M^3/\sqrt{Re_c}$, and it is suggested that only the data of Refs. 8-10 should be used to provide a comparison with the present inviscid theoretical results. Of the three sets of results presented by Schueler[8] for an axis position $x_0/c = 0.55$, only the highest Reynolds number tests show significant increase of damping with angle of attack. The magnitude of this increase is in broad agreement with the predictions of the current theory. Marshall's results,[9] although for a much smaller incidence range, also confirm this trend. For more rearward axis positions, the theory predicts much smaller increases in damping and for $x_0/c < 0.78$, a decrease in damping with increase in angle of attack is predicted. Recent experimental results by Qasrawi[10] show only a small increase in damping over the range $0 \leq x \leq 17$ deg for $x_0/c = 0.70$, thus supporting the predicted theoretical trends. As recognized by previous authors, see for example Ericsson,[11] the experimentally observed trends of the derivatives with angle of attack at larger values of the viscous interaction parameter $\bar{\chi}$ will result from a combination of nonlinear inviscid phenomena together with significant induced effects resulting from oscillatory transition point movement.

References

[1] East, R. A., Qasrawi, A. M., and Khalid, M., "An Experimental Study of the Hypersonic Dynamic Stability of Pitching Blunt Conical and Hyperballistic Shapes in a Short Running Time Facility," *AGARD Conference Proceedings*, No. 235, Nov. 1978.

[2] Khalid, M. and East, R. A., "Stability Derivatives of Blunt Slender Cones at High Mach Numbers," *Aeronautical Quarterly*, Vol. 30, Nov. 1979, pp. 559-590.

[3] Eggers, A. J. and Savin, R. C., "A Unified Two-Dimensional Approach to the Calculation of Three-Dimensional Hypersonic Flows with Application to Bodies of Revolution," NACA Rept. 1249, 1955.

[4] Stone, A. H., "On Supersonic Flow Past a Slightly Yawing Cone, II," *Journal of Math & Physics*, Vol. 30, 1951, pp. 200-213.

[5] Kopal, Z., "Tables of Supersonic Flow around Cones at Large Yaw," M.I.T. TR. No. 5, 1949.

[6] Taylor, G. I. and Maccoll, J. W., "Air Pressure on a Cone Moving at High Speeds," *Proceedings of the Royal Society of London, Series A*, Vol. 139, Feb. 1933, pp. 278-311.

[7] Khalid, M., "A Theoretical and Experimental Study of the Hypersonic Dynamic Stability of Blunt Axisymmetric Conical and Power Law Shapes," Ph.D. Thesis, University of Southampton, Hants, England, 1977.

[8] Schueler, C. J., "Dynamic Stability Results for a 10 deg Cone at Mach Numbers 0.8 to 20," AEDC-TDR 64-226, Dec. 1964.

[9] Marshall, L. A., "Hypersonic Dynamic Stability Summary," Pt. 1, Tech. Rept. AFFDL-TDR 66-149, Jan. 1967.

[10] Qasrawi, A. M., Unpublished data, University of Southampton, Hants, England, 1978.

[10] Ericsson, L. E., "Effect of Nose Bluntness, Angle of Attack, and Oscillation Amplitude on Hypersonic Unsteady Aerodynamics of Slender Cones," *AIAA Journal*, Vol. 9, Feb. 1971, pp. 297-304.

J80-240

Lifting and Nonlifting Kernel Functions for Cascade and Isolated Airfoils

00001 20016 90001

M. R. Chi*

Pratt & Whitney Aircraft, West Palm Beach, Fla.

THERE exists a considerable amount of similarity between the theoretical analyses of unsteady aerodynamics of cascade airfoils and isolated airfoils. The lifting and nonlifting kernel functions that relate the surface upwash distribution to the surface pressure load are of fundamental importance in linearized, unsteady, subsonic, potential-flow aerodynamics. It is the purpose of this Note to derive these kernel functions for two-dimensional flows in a unified fashion via the Fourier transform technique and specifically discuss the singular behavior of the nonlifting kernel function in the limit of zero Mach number. Because of its physical significance, the traveling wave-type of motion of the cascade blades will be considered.

Received Nov. 6, 1979; revision received Jan. 24, 1980.

Index categories: Subsonic Flow; Aerodynamics; Aeroelasticity and Hydroelasticity.

*Senior Research Engineer, Aeroelasticity and Vibration Group. Member AIAA.

Lifting Kernel Function

The subsonic lifting kernel function for two-dimensional linear cascade airfoils was evaluated previously by Fleeter[1] and Kaji and Okazaki.[2] Fleeter indicated the singular behavior of the lifting kernel function and computed the kernel function by inverting its Fourier transform numerically. Kaji and Okazaki derived an analytical expression for the lifting kernel function using the doublet singularity method. Their expression, as it stands, is inappropriate for zero Mach number flows. In this paper, a different expression for the lifting kernel is derived by analytically inverting Fleeter's Fourier integral. The resulting expression is valid for all subsonic Mach numbers, including zero.

Following Fleeter,[1] the lifting kernel function $K(x)$ for harmonic motions with frequency ω in a uniform stream with Mach number M is the Fourier inversion of K^*, i.e.,

$$K(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}K^*e^{i\alpha x}\mathrm{d}\alpha \qquad (1)$$

where

$$K^*=\frac{i}{2}\frac{\mu\sin(\mu h)}{[\alpha+(\omega/U)][\cos(\mu h)-\cos(\alpha d-\sigma)]} \qquad (2)$$

and

$$\mu^2=\beta^2\left[\left(\frac{\omega M}{U\beta^2}\right)^2-\left(\alpha-\frac{\omega M^2}{U\beta^2}\right)^2\right]$$

In the preceding equation, σ is the interblade phase angle and $\beta=\sqrt{1-M^2}$. Figure 1 defines the cascade geometric parameters d and h. It is noted that the expression of K^* is for any arbitrary stagger angle, and Fleeter's expression[1] is for zero stagger angle only. Physically, K is the upwash velocity corresponding to a spatial impulse pressure differential across airfoil upper and lower surfaces.

To simplify the Fourier inversion,

$$\bar{\alpha}=\alpha-\frac{\omega}{U}\frac{M^2}{\beta^2}$$

is defined. Then from Eq. (1),

$$K(x)=\frac{i}{4\pi}\exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]\times\int_{-\infty}^{\infty}\frac{\mu\sin(\mu h)e^{i\bar{\alpha}x}\mathrm{d}\bar{\alpha}}{[\bar{\alpha}+(\omega/\beta^2U)][\cos(\mu h)-\cos(\bar{\alpha}d-\sigma')]} \qquad (3)$$

where

$$\sigma'=\sigma-\frac{M^2}{\beta^2}\cdot\frac{\omega d}{U}$$

and

$$\mu^2=-\beta^2\left[(\bar{\alpha}^2)-\left(\frac{\omega M}{U\beta^2}\right)^2\right] \qquad (4)$$

The poles of the integrand in the complex $\bar{\alpha}$ plane are all of first-order and are located at

$$-\frac{\omega}{U}\frac{1}{\beta^2}\ \text{and}\ \bar{\alpha}_n^{\pm}$$

where $\bar{\alpha}_n^{\pm}$ are the zeros of $[\cos(\mu h)-\cos(\bar{\alpha}d-\sigma')]$, and the superscripts $\pm$ refer to the upper (+) and lower (−) $\bar{\alpha}$ half-